MA 262 Sec 1.1. Intro to Diff Eq.

Dr Daniels

(Ordinary) Diff. Eq (ODE)

Ex
- $y' = 2x$
- $y' = ky$ } $\left(y' = \frac{dy}{dx}\right)$ k const
- $x'' - 3x' + x = \cos(t)$ $\left(x' = \frac{dx}{dt}\right)$
- $\frac{dP}{dt} = k.P(M-P)$ (k, M const)

ODE involves
- <u>one</u> independent variable
- some dependent variable/function and its derivatives.

"Partial" Diff Eq (PDE) Example

x = position
t = time

$u(x,t)$ = heat @ position x, time t

"Heat Equation" $\frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial t} = 0$ is a PDE

<u>Idea</u>: Diff eq. is like a "template"/"model" you can plug a function in to, to check if the function "satisfies the equation": if equality is TRUE upon plugging in function.

ODE $\underline{y' = 7y}$

If $y = \sin(7x)$ then $y' = 7\cos(7x)$,

$\longrightarrow$ plug in to $\{y' = 7y\} \longrightarrow 7\cos(7x) \overset{?}{=} 7 \cdot \sin(7x)$ X

FALSE

$\Rightarrow y = \sin(7x)$ does not satisfy the ODE $\{y' = 7y\}$.

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If $y = e^{7x}$, then $y' = 7e^{7x} = 7 \cdot y$, so clearly $y = e^{7x}$ does satisfy the ODE $\{y' = 7y\}$

☆ "$y = f(x)$ (does not) satisfies [ODE]" $\longleftrightarrow$ "$y = f(x)$ is (not) a solution to [ODE]"

Ex: Is $y = e^x$ a solution of the ODE $\{y' = 12x^2(y^2+1)\}$?

$y = e^x$
$y' = e^x$ $\longrightarrow$ plug-in $e^x \overset{?}{=} 12x^2(e^{2x}+1)$? No

$y = e^x$ is not a solution.

Is $y = \tan(4x^3 + C)$ (c const) a solution (to [ODE])?

$y' = \sec^2(4x^3 + c) \cdot (12x^2)$

plug in

$$\cancel{12x^2}\sec^2(4x^3+C) \overset{?}{=} \cancel{12x^2}\left(\tan^2(4x^3+C) + 1\right)$$

$(\sec^2 x = \tan^2 x + 1)$

$$\sec^2(4x^3+C) = \tan^2(4x^3+C) + 1 \checkmark$$

$\Rightarrow$ is a solution for the ODE

★ Diff Eqs can have many solutions.

- Ex: $y = x^2 + C$ (C const.) is a solution to $\{y' = 2x\}$, regardless of value of C.

  ($y = x^2$, $y = x^2 - 3$, $y = x^2 + \pi$, ... all solutions)

Can think general "template function" $y = x^2 + C$ describes an infinite family of solutions to ODE $\{y' = 2x\}$

These "solution templates" which involve an unknown const (usually C) are called the/a general solution(s) [of the ODE]

- $y = x^2 + C$ is a general solution for $\{y' = 2x\}$
- $y = \tan(4x^3 + C)$ " " " " for $\{y' = 12x^2(y^2+1)\}$
- $y = Ce^{7x}$ is a general solution for $\{y' = 7y\}$

  → $y = Ce^{7x}$
  $y' = 7 \cdot Ce^{7x}$ ⟶ plug in $7 \cdot Ce^{7x} = 7 \cdot Ce^{7x}$ ✓

Newton's 2nd Law: If $x(t)$ is descibing motion of a physical thing, then $x(t)$ must satisfy equation $\{mx'' = F\}$